TWO DIMENSIONAL OPTIMAL CUTTING PROBLEM

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Abstract

At the 120-th European Study Group with Industry engineering company "STOBET" Ltd set the problem of optimizing the process of arranging plates with different shapes on a steel sheet. This report describes the methods proposed by the study group, and the conclusions drawn from the study of these approaches.

Key words: Two dimensional cutting stock problem, genetic algorithm, greedy algorithm

1. Introduction

Engineering company "STOBET" Ltd creates projects for buildings and structures.

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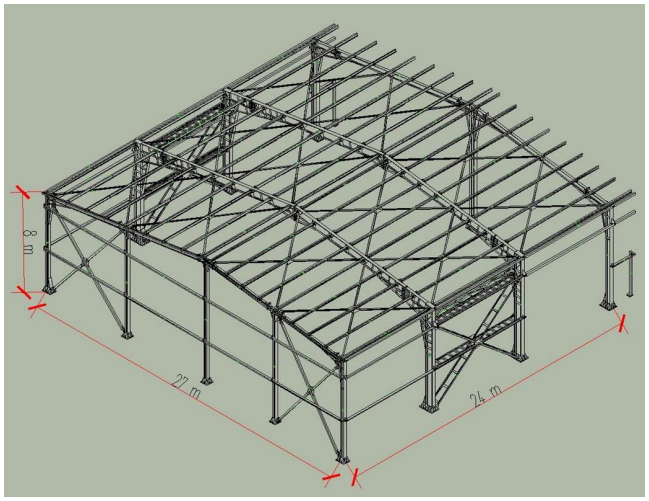

Figure 1. Steel construction

Having a lot of such projects, the company wants to improve the quality of the product and to reduce the price of the process of its realization. "STOBET" Ltd uses software for design of steel constructions. The software produces drawings of the different needed plates. After making all drawings, the task is to deploy optimally all plates on a minimal number of standard steel sheets.

1.1. Description of the problem

A standard steel sheet with dimensions $X = 1500$ mm, $Y = 12000$ mm is available. On each such sheet we have to locate maximal number of plates of different shapes and dimensions. The contour of a plate might be square, rectangle, trapeze or any non-crossing closed polygon, consisting of line segments only.

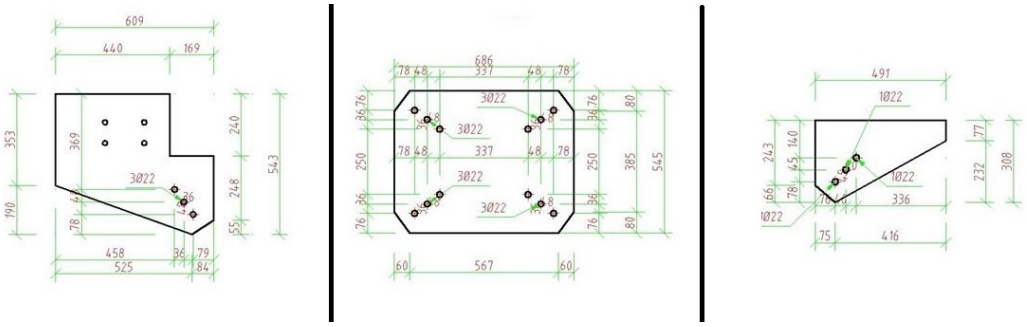

Figure 2. Examples of different plate's shapes

For any plate only the coordinates of its vertices are known. Let, for example, are given n plates with their coordinates:

- **Plate 1:** $(x_{11}, y_{11}; x_{12}, y_{12}; \ldots; x_{1i_1}, y_{1i_1})$
- **Plate 2:** $(x_{21}, y_{21}; x_{22}, y_{22}; \ldots; x_{2i_2}, y_{2i_2})$
- ...
- **Plate *n*:** $(x_{n1}, y_{n1}; x_{n2}, y_{n2}; \ldots; x_{ni_n}, y_{ni_n})$

The numbers $k_1, k_2, \ldots, k_n$ of the pieces of plates $1, 2, \ldots, n$ needed for the construction are given as well.

In the process of optimization some plates can be rotated for best fit. The aim is to locate the small plates on the given big steel sheet with minimal waste. For cutting a plate a cut of 5 mm has to be provided.

2. The approaches

Three different approaches to the two dimensional cutting problem in hand are considered by the group.

2.1. A Genetic algorithm

For each polygonal plate a rectangle (box) covering it is considered. The box is easily determined by the coordinates of its lower left vertex $(x_{\min}, y_{\min})$ and the coordinates of its upper right vertex $(x_{\max}, y_{\max})$, where $x_{\min}$ is the minimal x-coordinate of the plate, etc. The waste from every box is calculated as the difference between the box area and the polygon area. Then we use the genetic algorithm developed in [1] to deploy boxes on the steel sheet.

Genetic algorithm (GA) is a robust adaptive optimization method based on biological principles. A population of strings representing possible problem solutions is maintained. Search proceeds by recombining strings in the population. The theoretical foundations of genetic algorithms are based on the notion that selective reproduction and recombination of binary strings changes the sampling rate of hyperplanes in the search space so as to reflect the average fitness of strings that reside in any particular hyperplane. Thus, genetic algorithms need not search along the contours of the function being optimized and tend not to become trapped in local minima [4]. Some modifications of the standard form of the GA are accomplished and described in details in Final Reports of ESGI'113 [1].

The total waste is the sum of waste for each box and the waste obtained after application of the genetic algorithm.

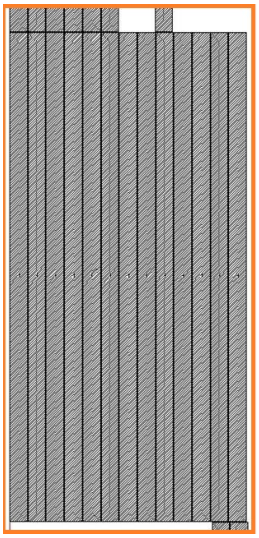

Figure 3. Example of a solution obtained by applying the genetic algorithm

2.2. Using Matching plates

The second approach is to arrange first several identical plates in series and after that put the series in boxes. Thus, we optimize the wastage coming from boxes. From the other hand, the resulting boxes might become big and the task to find enough space for putting the big box on the steel sheet becomes more difficult.

2.3. A Greedy algorithm

For each plate we calculate the area of the polygon and we sort plates in decreasing order by their area. The idea is to put the plates on the steel sheet in such a way that its width to be filled first.

We start by considering the contour of the steel sheet as negatively oriented (in counterclockwise direction). As a result we have a sequence of four points describing the negatively oriented contour of the sheet. We orient the contours of all plates positively (in clockwise direction). Calculating the oriented surface of each plate we obtain the orientation of the contour described by the sequence of vertexes of the plate. If necessary, we easily reverse the orientation by rearranging the vertex in opposite way. Finally, we have a sequence of points, describing the contour of the plate that is positively oriented.

We start by adjusting the plate with the biggest area into the lower left corner of the steel sheet. The plate can be rotated for best fit. We "cut" the plate. The

result is a sequence of points describing the rest of the sheet available for cutting which contour is negatively oriented. On the lower left corner of the rest we put the first plated of the ordered list that fits. If no such plate exists this corner is eliminated for further considerations, and we continue with the following lower left corner of the sheet. The sheet is completely flared when it has no corner in which a plate from the list of non-cut plates could be adjusted.

3. Results

Some experimental tests with real data were made using the suggested modified genetic algorithm. The total waste was decreased to 19%.

For a concrete example it was applied a genetic algorithm with two-dimensional topology of polygons. The result was worse. The total wastage of the steel sheet was 21.30%.

We have some difficulties to implement the greedy algorithm.

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