Laboratory calibration of MEMS rate sensors

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Abstract

In this paper, we suggest a simple algorithm for calibrating microelectromechanical systems (MEMS) accelerometers. We use a classical relation, depending on 12 parameters, between the raw output from the sensors and the calibrated data, as well as a more complicated relation, derived from physical and geometrical considerations. We obtain the calibration parameters by formulating and solving a least-squares problem. Results of numerical experiments are shown to validate the proposed algorithm.

Key words: MEMS accelerometers, sensors, calibration algorithms, parameter identification, reverse problem.

1. Introduction

In directional drilling, the orientation of the borehole is determined by measuring three angles—toolface, inclination, and azimuth angles (see Fig. 1). In the present work, we are interested in the problem of measuring the first two of them.

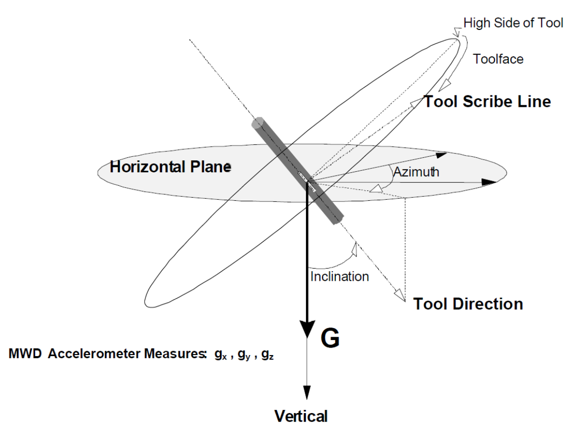

Figure 1: Toolface, inclination, and azimuth angle. [3]

As far as miniature dimensions and full measurement range are concerned, the most advantageous solution is a sensor made of commercial microelectromechanical systems (MEMS) accelerometers, which employs measurements of the three Cartesian components of the vector of gravitational acceleration [4].

Let us denote the acceleration vector, acting on a three-axial accelerometer sensor, with

$$\mathbf{a} = (a_x, a_y, a_z)^T.$$

When the device is in still position, it should be measuring only the gravitational acceleration. Then, using simple geometric considerations, the toolface (tf) and inclination ($incl$) can be easily computed, as follows:

$$tf(\mathbf{a}) = \begin{cases} \frac{360^\circ}{2\pi} atan2(a_y, a_x) & \text{if } atan2(a_y, a_x) \geq 0, \\ 360^\circ + \frac{360^\circ}{2\pi} atan2(a_y, a_x) & \text{otherwise,} \end{cases} \tag{1}$$

where

$$atan2(x, y) := \begin{cases} \arctan(\frac{y}{x}) & \text{if } x > 0, \\ \arctan(\frac{y}{x}) + \pi & \text{if } x < 0 \text{ and } y \geq 0, \\ \arctan(\frac{y}{x}) - \pi & \text{if } x < 0 \text{ and } y < 0, \\ +\frac{\pi}{2} & \text{if } x = 0 \text{ and } y > 0, \\ -\frac{\pi}{2} & \text{if } x = 0 \text{ and } y < 0, \\ \text{undefined} & \text{if } x = 0 \text{ and } y = 0. \end{cases}$$

The latter function computes the angle, which the segment through the point (x, y) and the origin makes with the positive x-semi-axis.

Let us remark that when using formula (1) in computer arithmetics, it is wise to substitute the condition for the first case on the right-hand side with $atan2(a_y, a_x) \geq -\varepsilon$ for some $\varepsilon > 0$, in order to avoid large errors for near-zero angles.

The inclination is given by

$$incl(\mathbf{a}) = \begin{cases} 90^\circ - \frac{360^\circ}{2\pi} \arcsin \sqrt{\frac{a_x^2 + a_y^2}{a_x^2 + a_y^2 + a_z^2}} & \text{if } 90^\circ - \frac{360^\circ}{2\pi} \arccos \frac{a_z}{\sqrt{a_x^2 + a_y^2 + a_z^2}} > 60^\circ, \\ 90^\circ - \frac{360^\circ}{2\pi} \arccos \frac{a_z}{\sqrt{a_x^2 + a_y^2 + a_z^2}} & \text{otherwise.} \end{cases} \tag{2}$$

It is well-known, however, that MEMS accelerometers are subject to different sources of error that should be accounted for, before they can be used in practice. There are two main types of errors—deterministic and random.

The deterministic error sources include the bias (offset) and the scale factor errors [2]. Another issue lies in the fact that formulae (1) and (2) are only valid if the three axes of the sensor are perfectly orthogonal. This, however, can never be the case in practice. The random errors include bias-drifts or scale factor drifts, and the rate at which these errors change with time. Furthermore, all the errors are sensitive to different environmental factors, especially to temperature variations [1].

In the present work, we are interested in the initial calibration in laboratory conditions of the deterministic sources of error. More specifically, we suggest an algorithm for compensating the errors due to nonorthogonalities, shifts and scale factors.

Apart from this, random errors should be accounted for by using an appropriate stochastic model. Many authors suggest using Alan variance as a tool for studying those errors [6]. Also, a temperature model should be used to account for the temperature variations.

2. Calibration approaches

As formulated, e.g., by Aggarwal et al. (2008), calibration is the process of comparing instrument outputs with known reference information and determining the coefficients that force the output to agree with the reference information over a range of output values [2].

Thus, we can divide the problem into two sub-problems:

1. propose a model, relating the raw output data from the sensors to the "real" components of the acceleration vector with respect to some orthonormal coordinate system;

2. determine the parameters in the above model by formulating and solving an appropriate minimization problem.

We shall now consecutively consider those two steps.

2.1 Relation between the measured and calibrated data

We assume the following classical linear relation between the raw data, $\hat{\mathbf{a}} = (\hat{a}_x, \hat{a}_y, \hat{a}_z)^T$, and the calibrated data, $\mathbf{a} = (a_x, a_y, a_z)^T$:

$$\mathbf{a} = \mathbf{M}.\hat{\mathbf{a}} + \mathbf{b}. \qquad (3)$$

In the latter, $\mathbf{M}$ is a full 3×3 matrix,

$$\mathbf{M} = \begin{bmatrix} m_{xx} & m_{xy} & m_{xz} \\ m_{yx} & m_{yy} & m_{yz} \\ m_{zx} & m_{zy} & m_{zz} \end{bmatrix},$$

wherein the diagonal elements account for the scale factors and the off-diagonal elements—for the errors due to non-orhogonalities. The vector

$$\mathbf{b} = \begin{bmatrix} b_x \\ b_y \\ b_z \end{bmatrix}$$

contains the offsets.

Another approach is to derive a more accurate form of the matrix M, using physical and geometrical considerations. It can be shown (see [5]) that

$$\mathbf{M} = \overline{\mathbf{T}}^{orth}\overline{\mathbf{T}}\mathbf{T}, \mathbf{b} = \overline{\mathbf{T}}^{orth}\overline{\mathbf{T}}\mathbf{T}\overline{\mathbf{b}} \tag{4}$$

where

$$\mathbf{T} = \begin{bmatrix} t_x & 0 & 0 \\ 0 & t_y & 0 \\ 0 & 0 & t_z \end{bmatrix}, \ \overline{\mathbf{b}} = \begin{bmatrix} b_x \\ b_y \\ b_z \end{bmatrix},$$

$$\overline{\mathbf{T}} = \frac{1}{den} \begin{bmatrix} -\sin^2\theta & \cos\phi - \cos\theta\cos\psi & \cos\psi - \cos\phi\cos\theta \\ \cos\phi - \cos\theta\cos\psi & -\sin^2\psi & \cos\theta - \cos\phi\cos\psi \\ \cos\psi - \cos\phi\cos\theta & \cos\theta - \cos\phi\cos\psi & -\sin^2\phi \end{bmatrix},$$

$$den = -1 + \cos^2\phi + \cos^2\psi + \cos^2\theta - 2\cos\phi\cos\psi\cos\theta,$$

$$\overline{\mathbf{T}}^{orth} = \begin{bmatrix} 1 & \cos\phi & \cos\psi \\ 0 & \sin\phi & \dfrac{\cos\theta - \cos\phi\cos\psi}{\sin\phi} \\ 0 & 0 & \dfrac{\sqrt{1 - \cos^2\phi - \cos^2\psi - \cos^2\theta + 2\cos\phi\cos\psi\cos\theta}}{\sin\phi} \end{bmatrix}.$$

In the above expressions, the 9 parameters have the following meaning. The three scaling coefficients are denoted by t_x, t_y, t_z, the offsets on the respective axes are denoted by b_x, b_y, b_z and φ, ψ, θ are the angles between the directions defined by the three axes of the MEMS accelerometer. Let us remark that those angles should be close to 90 degrees.

Both approaches have their advantages—in particular, relation (4) gives a more accurate description of the physical problem but it leads to a highly nonlinear minimization problem as can be seen from the next section. The model (3) suggests a more basic description of the relation between the measured and calibrated data but leads to a linear least squares problem with respect to the calibration parameters. We shall present numerical experiments with both models in order to compare their applicability and accuracy.

2.2 Definition of the objective function

Let us define an orthonormal coordinate system $Oxyz$ attached to the device. Let us assume we have obtained raw measurements $\hat{\mathbf{a}}_x$, $\hat{\mathbf{a}}_y$, $\hat{\mathbf{a}}_z$ of the device in positions where the gravitational acceleration is aligned with the x-, y-, and z-axes respectively. Further, let $\hat{\mathbf{a}}_i$, $i = \overline{1,s}$ denote measurements in s different (arbitrary) positions.

We suggest the following objective function that is to be minimized for obtaining the calibration parameters:

$$\begin{aligned} \varepsilon(\mathbf{M},\mathbf{b}) = &\left(\frac{\|\mathbf{M}.\hat{\mathbf{a}}_x + \mathbf{b} - (g,0,0)^T\|}{g}\right)^2 + \left(\frac{\|\mathbf{M}.\hat{\mathbf{a}}_y + \mathbf{b} - (0,g,0)^T\|}{g}\right)^2 \\ &+ \left(\frac{\|\mathbf{M}.\hat{\mathbf{a}}_z + \mathbf{b} - (0,0,g)^T\|}{g}\right)^2 + \sum_{i=1}^{s}\left(\frac{\|\mathbf{M}.\hat{\mathbf{a}}_i + \mathbf{b}\| - g}{g}\right)^2. \end{aligned} \tag{5}$$

Minimizing the latter with respect to the calibration parameters, we obtain the best values (in the least-squares sense) such that the computed acceleration is as close as possible to the gravitational acceleration and the coordinate axes are aligned with the natural axes of the device.

3. Numerical results

We are given raw accelerometer outputs obtained in 32 different positions of the sensor, see Table 1. For each position, the toolface and the inclination of the device are known. We use the first 20 of them (the shaded rows in the table) to calibrate the device and the rest is used as a test set. Let us remark that we only give here results from one data set, since other experiments we have carried lead to similar conclusions.

First, we calibrate the sensors, using model (3). For solving the minimization problem (5), we use the Wolfram Mathematica function *NMinimize*. The results are presented in Table 2. In all cases the computed toolface and inclination is within less than 2° from the real value. Let us remark that when the inclination

tf	incl	$\hat{a}_x$	$\hat{a}_y$	$\hat{a}_z$
0.00	0.00	-11718.00	966306.00	31738.00
10.00	0.00	-14648.00	938278.40	209862.40
20.00	0.00	-17577.80	891403.80	365428.20
30.00	0.00	-21386.40	815134.20	519431.40
40.00	0.00	-26171.40	710251.80	662986.20
50.00	0.00	-31151.40	583006.00	778122.80
60.00	0.00	-37109.00	440330.40	874997.00
70.00	0.00	-42675.20	277831.00	941500.80
80.00	0.00	-47851.00	112108.80	983199.80
90.00	0.00	-53124.40	58593.00	993258.60
50.00	30.00	271343.57	488907.43	780340.71
50.00	60.00	494627.00	249413.00	780759.00
50.00	90.00	566795.20	-59472.20	779294.00
50.00	120.00	474412.40	-366600.40	775680.80
50.00	150.00	239842.60	-586619.20	771286.80
50.00	180.00	-80956.40	-656833.60	766696.60
50.00	210.00	-378986.33	-561766.00	763750.50
50.00	240.00	-597068.40	-326463.80	763278.60
50.00	270.00	-668943.00	-8984.20	765623.00
50.00	300.00	-573533.40	296189.80	769626.60
50.00	330.00	-337498.60	513182.00	774020.80
0.00	330.00	-503904.00	852243.20	22851.00
0.00	300.00	-884177.20	491209.00	13671.00
0.00	270.00	-1028903.00	4979.80	7324.00
0.00	240.00	-917673.20	-493357.60	4296.40
0.00	210.00	-562888.40	-874118.40	5371.00
0.00	180.00	-69432.80	-1023629.60	9277.00
0.00	150.00	413377.60	-911911.40	16698.60
0.00	120.00	787302.20	-571872.80	24804.40
0.00	90.00	943681.83	-52815.33	31167.67
0.00	60.00	830889.33	433998.83	34504.33
0.00	30.00	483494.60	812399.40	33788.60

Table 1: Raw data measured by a three-axial accelerometer.

equals 90°, the toolface cannot be computed, because the function $tf(\mathbf{a})$ is not defined, when $a_x = a_y = 0$ holds true.

a_x	a_y	a_z	comp. tf	error	comp. incl	error
0.00000	9.81605	0.00108	0.000	0.000	0.006	0.006
-0.00187	9.60862	1.77483	-0.011	0.011	10.465	0.465
-0.00039	9.20464	3.32969	-0.002	0.002	19.887	0.113
0.00109	8.50745	4.87664	0.007	0.007	29.822	0.178
0.00040	7.52117	6.32738	0.003	0.003	40.073	0.073
0.00156	6.30094	7.50205	0.014	0.014	49.973	0.027
-0.00430	4.91951	8.49967	-0.050	0.050	59.938	0.062
-0.00343	3.32881	9.20115	-0.059	0.059	70.111	0.111
-0.00031	1.69614	9.65744	-0.010	0.010	80.039	0.039
0.00000	0.00108	9.80126	-	-	89.994	0.006
3.05564	5.48063	7.51727	29.141	0.859	50.147	0.147
5.36162	3.18410	7.55978	59.295	0.705	50.482	0.482
6.17725	0.13979	7.61671	88.704	1.296	50.950	0.950
5.34648	-2.95013	7.66863	118.889	1.111	51.470	1.470
3.06713	-5.22806	7.70505	149.601	0.399	51.809	1.809
-0.12023	-6.04927	7.71009	181.139	1.139	51.877	1.877
-3.13003	-5.21794	7.68696	210.958	0.958	51.636	1.636
-5.38272	-2.96114	7.64445	241.184	1.184	51.213	1.213
-6.19794	0.16909	7.59403	271.563	1.563	50.770	0.770
-5.33626	3.24088	7.54617	301.272	1.272	50.397	0.397
-3.04133	5.48927	7.51037	331.011	1.011	50.119	0.119
-4.88847	8.49236	-0.00786	330.074	0.074	0.046	0.046
-8.58255	4.75541	0.03191	298.990	1.010	0.186	0.186
-9.88254	-0.13606	0.10774	269.211	0.789	0.625	0.625
-8.61874	-5.04982	0.19352	239.633	0.367	1.110	1.110
-4.95456	-8.70087	0.26520	209.659	0.341	1.517	1.517
0.02633	-9.99923	0.29185	179.849	0.151	1.672	1.672
4.82159	-8.70299	0.28781	151.013	1.013	1.657	1.657
8.45846	-5.17762	0.24339	121.472	1.472	1.406	1.406
9.86501	0.04465	0.15817	89.741	0.259	0.919	0.919
8.58912	4.84337	0.07855	60.582	0.582	0.456	0.456
4.99960	8.47390	0.01028	30.541	0.541	0.060	0.060

Table 2: Results from the calibration, using the linear model (3).

Next, we compare the latter results with those, obtained using model (4), see Table 3. The obtained accuracy is very similar, even slightly worse. On the other hand, the model (4) leads to a much more complex nonlinear minimization problem to be solved. Further experiments should establish if there are cases when the seemingly more accurate model (4) should be preferred. The current state of our research, however, suggests that the basic relation (4) gives comparable results and, thus, should be preferred because of its simplicity.

4. Conclusions and discussion

We have proposed an algorithm for calibrating MEMS accelerometers in laboratory conditions that aims to compensate the main deterministic errors. Based on numerical experiments, we suggest that a linear relationship between the raw and the calibrated data, depending on 12 parameters, should be used. We obtain those parameters by solving a least-squares problem, comparing the output accelerometer vector to the gravitational acceleration.

If the inclination is close to 90°, however, the following issue might arise. In this case the components of the gravitational acceleration on the $x-$ and $y-$ axes are small. Thus, a large relative error in those components might have little effect on the objective function (5), but lead to a substantial error when computing the toolface with formula (1). One possible approach is to adjust the objective function by comparing the toolfaces and inclinations, computed with formulae (1) and (2).

Further, numerical methods should be proposed for the effective solution of the minimization problem. Also, one can consider how to choose the s different positions in order to obtain optimal (in some sense) results.

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a_x	a_y	a_z	comp. tf	error	comp. incl	error
0.00000	9.80610	0.00000	0.000	0.000	0.000	0.000
0.01071	9.58144	1.79767	0.064	0.064	10.626	0.626
0.02139	9.16502	3.37025	0.134	0.134	20.190	0.190
0.02965	8.45899	4.93054	0.201	0.201	30.237	0.237
0.03271	7.46843	6.38896	0.251	0.251	40.545	0.545
0.03350	6.24978	7.56370	0.307	0.307	50.433	0.433
0.02467	4.87386	8.55637	0.290	0.290	60.333	0.333
0.01843	3.29446	9.24534	0.320	0.320	70.387	0.387
0.01210	1.67631	9.68477	0.414	0.414	80.180	0.180
0.00000	0.00000	9.80610	-	-	90.000	0.000
3.03319	5.43059	7.55588	29.185	0.815	50.538	0.538
5.28494	3.15470	7.55705	59.166	0.834	50.839	0.839
6.06388	0.14401	7.56749	88.640	1.360	51.287	1.287
5.22186	-2.90706	7.57828	119.105	0.895	51.739	1.739
2.96037	-5.15165	7.59054	150.116	0.116	51.947	1.947
-0.18347	-5.95412	7.59552	181.765	1.765	51.894	1.894
-3.13954	-5.12407	7.59530	211.496	1.496	51.649	1.649
-5.33916	-2.88758	7.59335	241.594	1.594	51.362	1.362
-6.11696	0.20791	7.59076	271.947	1.947	51.120	1.120
-5.24463	3.24094	7.58371	301.714	1.714	50.890	0.890
-2.96804	5.45629	7.57160	331.455	1.455	50.636	0.636
-4.82237	8.51211	-0.01023	330.467	0.467	0.060	0.060
-8.48801	4.82994	-0.01057	299.641	0.359	0.062	0.062
-9.80610	0.00000	0.00000	270.000	0.000	0.000	0.000
-8.60074	-4.85926	0.01085	240.534	0.534	0.063	0.063
-5.02271	-8.47755	0.01687	210.646	0.646	0.098	0.098
-0.13020	-9.77458	0.00656	180.763	0.763	0.038	0.038
4.60010	-8.50730	0.00355	151.599	1.599	0.021	0.021
8.20770	-5.03397	-0.00369	121.522	1.522	0.022	0.022
9.63324	0.12261	-0.01938	89.271	0.729	0.115	0.115
8.41512	4.86818	-0.02560	59.950	0.050	0.151	0.151
4.91044	8.46595	-0.02874	30.115	0.115	0.168	0.168

Table 3: Results from the calibration, using the linear model (3).

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